

TRIGONOMETRIC RATIOS OF COMPOUND ANGLES

- 1) Show that $\tan(45^\circ + A) \cdot \tan(45^\circ - A) = 1$
- 2) Prove that $\sin^2 52 \frac{1}{2}^\circ - \sin^2 22 \frac{1}{2}^\circ = \frac{\sqrt{3}+1}{4\sqrt{2}}$
- 3) Prove that $\frac{\cos 27^\circ + \sin 27^\circ}{\cos 27^\circ - \sin 27^\circ} = \cot 18^\circ$ (or) $\tan 72^\circ$
- 4) If $A+B=45^\circ$, show that $(1+\tan A)(1+\tan B)=2$
- 5) Prove that $\cos 70^\circ \cos 10^\circ + \sin 70^\circ \sin 10^\circ = \frac{1}{2}$
- 6) Prove that $\frac{(\tan A + \sec A - 1) \cos A}{\sin A + \cosec A - 1} = \frac{1 + \sin A}{\cos A}$
- 7) Prove that $\sin^2 45^\circ - \sin^2 15^\circ = \frac{\sqrt{3}}{4}$.
- 8) Prove that $\sin^2 75^\circ - \cos^2 45^\circ = \frac{\sqrt{3}}{4}$
- 9) Prove that $\cos^2 48^\circ - \sin^2 12^\circ = \frac{\sqrt{5}+1}{8}$
- 10) Prove that $\cos^2 15^\circ - \cos^2 75^\circ = \frac{\sqrt{3}}{2}$.
- 11) If $\tan \theta = \sqrt{3}$ and $\tan \phi = 2 - \sqrt{3}$, show that $(\theta - \phi) = \frac{\pi}{4}$.
- 12) If $\sin A = \frac{3}{5}$, $\cos B = \frac{5}{13}$ find $\sin(A+B)$ Ans. $\frac{63}{65}$
- 13) Prove that $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$ or $\cot 34^\circ$
- 14) Prove that $\frac{\cos 37^\circ + \sin 37^\circ}{\cos 37^\circ - \sin 37^\circ} = \cot 8^\circ$
- 15) Prove that $\frac{\cos 19^\circ - \sin 19^\circ}{\cos 19^\circ + \sin 19^\circ} = \tan 26^\circ$
- 16) If $A+B = 135^\circ$, prove that $(1+\cot A)(1+\cot B)=2$
- 17) Prove that $\tan 8A - \tan 5A - \tan 3A = \tan 8A \cdot \tan 5A \cdot \tan 3A$
- 18) If $A+B+C = 90^\circ$, then prove that
 - (1) $\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A = 1$
 - (2) $\cot A + \cot B + \cot C = \cot A \cdot \cot B \cdot \cot C$
- 19) Prove that $\frac{\cos(\alpha-\beta)}{\cos} = \frac{1+\sin A}{\cos A}$

TRIGONOMETRIC (MULTIPLE AND SUB MULTIPLE ANGLES)

- 1) Prove that $(\cos A + \sin A)^2 = 1 + \sin^2 A$
- 2) If $\sin A = \frac{3}{5}$, find (i) $\sin 2A \cos 2A$ (ii) $\sin 3A$
- 3) Prove that

$$1 + \cos 2A = \cot A \sin 2A$$

$$\cos^4 A - \sin^4 A = \cos 2A.$$
- 4) Prove that
 - a. $\sin x \cdot \sin(60^\circ - x) \cdot \sin(60^\circ + x) = \frac{1}{4} \sin 3x$
 - b. $\cos x \cdot \cos(60^\circ + x) \cdot \cos(60^\circ - x) = \frac{1}{4} \cos 3x$
 - c. $\tan A \cdot \tan\left(\frac{\pi}{3} + A\right) \cdot \tan\left(\frac{\pi}{3} - A\right) = \tan 3a$
 - d. $\cot \theta - \cot 2\theta = \cosec 2\theta$
- 5) Prove that $\frac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$
- 6) Prove that $\frac{\tan 2\theta}{1 + \sec 2\theta} = \tan \theta$
- 7) Prove that $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$
- 8) Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta$
- 9) Prove that $\frac{\sin 2A}{1 + \cos 2\theta} \cdot \frac{\cos A}{1 + \cos A} = \tan \frac{A}{2}$
- 10) Prove that $\frac{\sin 2\theta}{1 + \cos 2\theta} \cdot \frac{\cos \theta}{1 + \cos \theta} = \tan \frac{\theta}{2}$
- 11) Prove that $\frac{\sin 2\theta}{1 - \cos 2\theta} \cdot \frac{1 - \cos \theta}{\cos \theta} = \tan \frac{\theta}{2}$
- 12) Prove that $\frac{\cosec \theta}{\cosec 3\theta} \cdot \frac{\sec \theta}{\sec 3\theta} = 2$
- 13) Prove that $\frac{\cos 2A}{1 - \sin 2A} = \cot(45^\circ - A)$
- 14) Prove that $\tan\left(\frac{\pi}{4} + A\right) \cdot \tan\left(\frac{\pi}{4} - A\right) = 2 \tan 2A$
- 15) Show that $\cot\left(\frac{\pi}{4} + A\right) + \cot\left(\frac{\pi}{4} - A\right) = 2 \sec 2A$
- 16) Show that $\frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta$
- 17) Prove that $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \cdot \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = 2 \tan 2\theta$
- 18) Show that $\cos^6 \theta - \sin^6 \theta = 1 - \frac{3}{4} \sin^2 2\theta$
- 19) Show that $\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$
- 20) Prove that $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$.
- 21) Show that $\cos 10^\circ \cos 50^\circ \cos 70^\circ = \frac{\sqrt{3}}{8}$.
- 22) Show that $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$
- 23) Show that $\cos 20^\circ \cos 30^\circ \cos 40^\circ \cos 80^\circ = \frac{\sqrt{3}}{16}$.
- 24) If $a \sin \alpha = b \cos \alpha$,
show that $a \cos 2\alpha + b \sin \alpha = a$.
- 25) Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$.

26) If $\tan a = \frac{1-\cos B}{\sin B}$, then prove that $\tan 2a = \tan b$

27) Express $\sin 5\theta$ in terms of $\sin \theta$ (or)

Show that $\sin 5\theta = 16\sin^2\theta - 20\sin^3\theta + 5\sin\theta$

28) If $x + \frac{1}{x} = 2\cos\theta$, then show that $x^2 + \frac{1}{x^2} = 2\cos 2\theta$

29) Prove that $\sin(60^\circ - A)\sin(60^\circ + A) = \frac{1}{4}[3 - 4\sin^2 A]$

30) Prove that $\tan\left(\frac{\pi}{4} + A\right)\tan\left(\frac{\pi}{4} - A\right) = 1$

TRANSFORMATION OF PRODUCTS AND SUMS

1) Prove that

a) $\cos(2x-3y) - \cos(3x-2y) = 2\sin\left(\frac{5x-5y}{2}\right)\sin\left(\frac{x+y}{2}\right)$

b) $\frac{\sin^2 A + \sin^2 B}{\sin A \cos A - \sin B \cos B} = \tan(A + B)$

c) $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{\sqrt{3}}{8}$

d) $\cos 20^\circ \cos 30^\circ \cos 40^\circ \cos 80^\circ = \frac{\sqrt{3}}{16}$

e) $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$

f) $\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ = \frac{1}{16}$

2) If $\sin x + \sin y = \frac{3}{4}$ and $\sin x - \sin y = \frac{2}{5}$

Prove that

$$8\cot\left(\frac{x-y}{2}\right) = 15\cot\left(\frac{x+y}{2}\right) \text{ (or)}$$

$$8\tan\left(\frac{x+y}{2}\right) = 15\tan\left(\frac{x-y}{2}\right)$$

3) If $\sin x + \sin y = a$ and $\cos x - \cos y = b$

Show that

$$\sin(x+y) = \frac{2ab}{a^2+b^2} \text{ and } \cos(x+y) = \frac{b^2-a^2}{a^2+b^2}.$$

4) $\sin \alpha + \sin \beta = a$ and $\cos \alpha - \cos \beta = b$, then prove

$$\text{that } \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{a}{b}$$

5) Show that $b \tan \alpha = a \tan \beta$, if $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a+b}{a-b}$.

6) If $\tan(\alpha+\beta) = 3\tan \alpha$, then

show that

$$\sin(2\alpha+\beta) + \sin \alpha = 2\sin 2\beta.$$

7) $A+B+C=90^\circ$, show that

$$\begin{aligned} \tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A &= 1 \\ \cot A + \cot B + \cot C &= \cot A \cdot \cot B \cdot \cot C \end{aligned}$$

INVERSE TRIGONOMETRIC FUNCTIONS

1) Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

2) Prove that $\sec^{-1} x + \cosec^{-1} x = \frac{\pi}{2}$

3) Prove that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{2}\right)$.

4) Prove that $\tan^{-1}\left(\frac{2}{7}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{17}{33}\right)$.

5) Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$.

6) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$.

Show that $xy + yz + zx = 1$

Then prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

7) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$,

Prove that $x + y + z = xyz$.

8) Prove that $\frac{\cos 7A}{\sec A} - \frac{\sin 7A}{\cosec A} = \cos 8A$

9) $\frac{\sin 3A \sin 7A + \sin A \sin 11A}{\sin 3A \cos 7A + \sin A \cos 11A} = \tan 8A$

10) $\frac{\sin 7A + \sin 17A}{\cos 7A + \cos 17A} = \tan 12A$

11) $\frac{\cos 7A + \cos 5A}{\sin 7A + \sin 5A} = \cot 6A$

12) $\frac{\sin 15A + \sin 5A}{\cos 15A + \cos 5A} = \tan 10A$

13) $\frac{\sin 8A + \sin 6A}{\cos 8A + \cos 6A} = \tan 7A$

14) $\frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A - \cos 3A - \cos 4A} = \cot A$

15) If $A+B+C=\pi$ (or 180°), then prove that following identities

a) $\sin 2A + \sin 2B + \sin 2C = 4\sin A \cdot \sin B \cdot \sin C$

b) $\sin 2A - \sin 2B + \sin 2C = 4\cos A \cdot \sin B \cdot \cos C$

c) $\sin 2A + \sin 2B - \sin 2C = 4\cos A \cdot \cos B \cdot \sin C$

d) $\sin 2A - \sin 2B - \sin 2C = -4\sin A \cdot \cos B \cdot \cos C$

e) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cdot \cos B \cdot \cos C$

f) $\cos 2A - \cos 2B + \cos 2C = 1 - 4\sin A \cdot \cos B \cdot \sin C$

g) $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A \cdot \sin B \cdot \sin C$

h) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

i) $\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

j) $\tan^2 \frac{A}{2} \tan^2 \frac{B}{2} + \tan^2 \frac{B}{2} \tan^2 \frac{C}{2} + \tan^2 \frac{C}{2} \tan^2 \frac{A}{2} = 1$

k) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cdot \cos B \cdot \cos C$

l) $\sin^2 A - \sin^2 B - \sin^2 C = -2\cos A \cdot \sin B \cdot \sin C$

16) if $A+B+C=270^\circ$, then prove

$\cos 2A + \cos 2B + \cos 2C = 1 - 4\sin A \cdot \sin B \cdot \sin C$

17) If $A+B+C = \frac{\pi}{2}$ prove that,

$\sin 2A + \sin 2B + \sin 2C = 4\cos A \cdot \cos B \cdot \cos C$

8) show that

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = 2\tan^{-1}\left(\frac{x+y}{1-xy}\right).$$

9) Solve $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$.

10) Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$.

11) Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \cos^{-1}\left(\frac{16}{65}\right)$.

12) Prove that $\cot^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{5}{12}\right) = \cos^{-1}\left(\frac{63}{65}\right)$.

13) Prove that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{4}{7}\right)$.

14) Prove that $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$.

15) Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{\pi}{4}\right)$.

- 16) Prove that $\sec^{-1}\left(\frac{5}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right) = \cot^{-1}\left(\frac{33}{56}\right)$.
- 17) Prove that $\tan^{-1}x + \cot^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4}$.
- 18) $\tan^{-1}\left(\frac{2}{3}\right) + \cot^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{17}{6}\right)$ or $\cot^{-1}\left(\frac{6}{17}\right)$
- 19) $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{9}\right) = \frac{\pi}{4}$
- 20) $\tan^{-1}(n) - \tan^{-1}(n^2+n+1) + \cot^{-1}(n+1) = 0$.
- 21) $\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cot^{-1}\left(\frac{11}{27}\right)$.
- 22) $\tan^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right) = \cot^{-1}\left(\frac{11}{27}\right)$.
- 23) $\cosec^{-1}\left(\frac{5}{4}\right) + 2 \cot^{-1}3 = \left(\frac{\pi}{2}\right)$.
- 24) $\sin\left[\cot^{-1}\frac{23}{7} + 2\tan^{-1}\frac{1}{4}\right] = \frac{1}{\sqrt{2}}0$.

- 25) $\cos^{-1}\frac{x}{a} + \cos^{-1}\left(\frac{y}{b}\right) = 0$,
Show that $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos\theta + \frac{y^2}{b^2} = \sin^2\theta$.
- 26) If $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = 0$, prove that
 $9x^2 - 12xycos\theta + 4y^2 = 36\sin^2\theta$.
- 27) Solve $\sin^{-1}(1-x) + \cos^{-1}x = \sin^{-1}(3x-2)$.
- 28) $\tan^{-1}\left(\frac{x}{1+x}\right) + \tan^{-1}\left(\frac{x}{1-x}\right) = \tan^{-1}2$.
- 29) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{1}{2}\right)$
- 30) $\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) + \sec^{-1}\left(\frac{1+b^2}{1-b^2}\right) = 2\tan^{-1}x$.

TRIGONOMETRIC EQUATIONS

- 1) Solve $\tan^2\theta = 3$
 $4 \sin^2 A - 3 = 0$
 Ans. (i) $\theta = n\pi - \frac{\pi}{6}$, $\theta = n\pi \pm \frac{\pi}{6}$
 Ans. (ii) $n\pi + (-1)^n \frac{\pi}{6}$, $n\pi \pm (-1)^n \frac{\pi}{6}$
- 2) Solve $2\sin^2 x - 5 \cos x = 4$.
 Ans. $\cos x = -2, \cos x = -1/2$
- 3) Solve $\sin x - \sin 2x + \sin 3x - \sin 4x = 0$
- 4) Solve $\sin 4\theta - \sin 3\theta + \sin 2\theta - \sin \theta = 0$
- 5) Solve $8 \sin^3 \theta = \sin 3\theta$
- 6) Solve $a \tan x + b \sec x = c$, show that $\tan(\alpha+\beta) = \frac{2ac}{a^2-c^2}$.
- 7) Solve $\cos \theta + \sqrt{3} \sin \theta = 1$
 Ans. $\theta = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{3}$
- 8) Solve $\cos x + \sin x = \cos 2x$
- 9) Solve $\cos 8\theta + \cos 2\theta - \cos 5\theta = 0$
 Ans. case(i) $\frac{\pi}{2}$ case(ii) $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$
- 10) Solve $\sin 6\theta \cos 2\theta = \sin 5\theta \cos \theta = 0$
 Ans. case(i) $\frac{2n\pi}{7} \pm \frac{\pi}{14}$ case(ii) $n\pi$

11) Solve the following trigonometric equations.

- a. $4 + \cos\theta - 6 \sin^2\theta = 0$
 Ans. $\theta = 2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \cos^{-1}\left(-\frac{2}{3}\right)$
- b. $2 \sin^2\theta + 3 \cos\theta - 3 = 0$
 Ans. $\theta = 2n\pi, \theta = 2n\pi \pm \frac{\pi}{3}$
- c. $2 \cos^2\theta + 3 \sin\theta = 0$
 Ans. $\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$
- d. $\sin^2\theta + \sin\theta - 1 = 0$
 Ans. $\theta = n\pi + (-1)^n \frac{\pi}{6}, \theta = n\pi + (-1)^n \left(\frac{\pi}{2}\right)$
- e. $2 \sin^2\theta = 1 + \cos\theta$
 Ans. $\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$
- f. $\cos 3x + 8 \cos^3 x = 0$
 Ans. $\theta = n\pi \pm \frac{\pi}{3}, \theta = (2n+1)\frac{\pi}{3}$
- g. $\sin\theta + \sin 2\theta + \sin 3\theta = 0$
- h. $\cos\theta + \cos 5\theta = \cos 3\theta$
 Ans. $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{6}, n\pi \pm \frac{\pi}{6}$
- i. $\sin 5\theta + \sin\theta = \sin 3\theta$
 Ans. $\theta = \frac{n\pi}{3}, n\pi \pm \frac{\pi}{6}$
- j. $\tan^2(\theta/2) + \sec(\theta/2) = 1$

- k. $\sin^3\theta + \sin^2\theta \cos\theta - \cos^2\theta \sin\theta - \cos^3\theta = 0$
- l. $\tan\theta + \tan 2\theta + \tan 3\theta = 0$ Ans. $\theta = \frac{n\pi}{3}, n\pi \pm \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$
- m. $\sin\theta \sin 3\theta = \sin 5\theta \sin 7\theta$
- n. $\cos 2\theta \cos 4\theta = \cos 6\theta \cos 8\theta$ Ans. $\theta = \frac{n\pi}{10}, \theta = \frac{n\pi}{4}$
- o. $\sin 6\theta \cos 2\theta = \sin 5\theta \cos\theta$ Ans. $\theta = n\pi, \frac{2n\pi}{7} \pm \frac{\pi}{14}$
- p. $\sqrt{3} \cos\theta + \sin\theta = \sqrt{2}$ Ans. $\theta = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{3}$
- q. $2 \operatorname{cosec}\theta - \cot\theta = \sqrt{3}$
- r. $2 \cos\theta + 4 \sin\theta + \cos 2\theta - \sin 2\theta = 3$ Ans. $\theta = n\pi + (-1)^n \frac{\pi}{2}, \theta = 2n\pi + \frac{\pi}{2}$

SOLUTIONS OF TRIANGLES

SOLVE THE TRIANGLE ΔABC

- a) $A=2, b=\sqrt{2}, c=\sqrt{3}+1$ Ans. $A=45^\circ, b=30^\circ, c=105^\circ$ Ans. $A=66^\circ 42', b=7.71, c=10.12$
- b) $A=13, b=14, c=15$ Ans. $A=53^\circ 13', b=59^\circ 49', c=67^\circ 38'$ j) $b=34, A=31^\circ 32', c=98^\circ 51'$ Ans. $A=23.34, b=49^\circ 37', c=44.1$
- c) $A=2, b=2\sqrt{3}, c=4$ Ans. $A=30^\circ, b=60^\circ, c=90^\circ$ k) $A=2, A=30^\circ, c=30^\circ$ Ans. $A=\frac{2}{\sqrt{3}}, b=120^\circ, c=2$
- d) $A=1, b=2, c=\sqrt{3}$ Ans. $A=30^\circ, b=90^\circ, c=60^\circ$ l) $c=\sqrt{2}, A=45^\circ, b=105^\circ$ Ans. $A=2, b=\sqrt{3+1}, c=30^\circ$
- e) $A=1, b=\sqrt{3}, c=2$ Ans. $A=30^\circ, b=60^\circ, c=90^\circ$ m) $c=3.78, A=37^\circ, b=84^\circ$ Ans. $A=2.654, b=4.385, c=59^\circ$
- f) $b=1, c=\sqrt{3}, A=30^\circ$ Ans. $A=1, b=30^\circ, c=120^\circ$ n) $A=9, b=65^\circ, c=15^\circ$ Ans. $A=100^\circ, b=8.3, c=2.4$
- g) $A=2, c=\sqrt{3}+1, b=60^\circ$ Ans. $b=\sqrt{6}, b=45^\circ, c=75^\circ$ o) $A=18, A=43^\circ, b=64^\circ$ Ans. $c=25.24, b=23.73, c=73^\circ$
- h) $A=2, c=\sqrt{6}, b=60^\circ$ Ans. $A=1, b=30^\circ, c=120^\circ$ p) $A=2, b=\sqrt{6}, b=120^\circ$ Ans. $A=45^\circ, c=15^\circ, c=\sqrt{3-1}$
- i) $A=10, b=45^\circ, c=68^\circ$ Ans. $c=10\sqrt{2}, A=30^\circ$ q) $A=10, b=10\sqrt{2}, A=30^\circ$ Ans. $c=19.32, b=105^\circ, c=45^\circ$

COMPLEX NUMBERS

1. Express the following in the form $a+ib$;

- a) $\frac{1+i}{1-i}$ Ans. $0+i(1)$
- b) $\left(\frac{3+i}{3-i}\right)^2$ Ans. $A = \frac{7}{25}, b = \frac{24}{25}$
- c) $\frac{(1+i)(1+3i)}{(1+5i)}$ Ans. $\frac{9}{17} + \frac{7}{13}i$
- d) $\frac{1+8i}{5-2i}$ Ans. $-\frac{11}{29} + i\left(\frac{42}{39}\right)$
- e) $\frac{(1+i)(2+i)}{(3+i)}$ Ans. $\frac{3}{5} + i\left(\frac{4}{5}\right)$

f) $\frac{2-3i}{3+4i}$ Ans. $-\frac{6}{25} - i\left(\frac{17}{25}\right)$

1) Find the conjugate of the following complex numbers $(2+5i)$ $(-4+6i)$ Ans. $-38+8i$

2) Find the Additive inverse of

$(2+3i)(4+2i)$ Ans. $2+i16$ is $-2-i16$

$\frac{2i}{1-2i}$ Ans. $\frac{2}{5}i - \frac{4}{5}$ is $\frac{2}{5}i + \frac{4}{5}$

3) Find the multiplicative inverse of

$2+3i$ Ans. $\frac{2-3i}{13}$

$-14+8i$

4) Express the following in the modulus Amplitude form (polAr form)

$1+i\sqrt{3}$ $\sqrt{3}+i$

5) find the real And imaginary parts of the following

$\frac{4+2i}{1-2i}$ Ans. 0, 2

$\frac{2}{5}$ Ans. $\frac{13}{65}, \frac{26}{65}$

$\left(\frac{1+i}{1-i}\right)^5$ Ans. 0, 1

6) Find the modulus of the following complex numbers

$(3-4i)(2-3i)$ Ans. 2

$\frac{(3+2i)^2}{4-3i}$ Ans. $\frac{13}{5}$

$\frac{3-4i}{5+7i}$ Ans. $\frac{5}{\sqrt{74}}$

$\frac{3-4i}{5+7i}$ Ans. $\frac{1}{\sqrt{2}}$

7) Find the Additive And multiplicative inverse of the following complex numbers

$4+3i$ Ans. $-4-3i, \frac{4}{25} - i\frac{3}{25}$

$\frac{9}{2+i\sqrt{5}}$ Ans. $-2+i\sqrt{5}, \frac{2}{9} + i\frac{\sqrt{5}}{9}$

$\frac{9}{2-i\sqrt{5}}$ Ans. $-2-i\sqrt{5}, \frac{2}{9} - i\frac{\sqrt{5}}{9}$

$\frac{2i}{1-2i}$ Ans. $\frac{4-2i}{5}, -1\frac{i}{2}$

8) Find the $z\bar{z}$, if $z=1+i4\sqrt{3}$ Ans. 49

9) Express the following in the modulus-Amplitude form

$\sqrt{3+i}$ Ans. $2 \left[\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right]$

$3+4i$ Ans. $5 \left[\cos\left(\tan^{-1}\frac{4}{3}\right) + i \sin\left(\tan^{-1}\frac{4}{3}\right) \right]$