2. DEFINITE INTEGRALS

- **BASIC DEFINITE INTEGRATION PROBLEMS**

1. \( \int_{-2}^{2} (x^2 - 4x + 3) \, dx \)
2. \( \int_{0}^{\pi} \cos^2 x \, dx \)
3. \( \int_{1/2}^{1} \frac{1}{\sqrt{1-x^2}} \, dx \)
4. \( \int_{0}^{\pi/2} \sqrt{1 + \sin 2x} \, dx \)
5. \( \int_{0}^{\pi/4} \tan^2 x \, dx \)
6. \( \int_{0}^{\pi/2} \sin^3 x \, dx \)
7. \( \int_{0}^{\pi/4} \sec^2 x \, dx \)

- **PROPERTY BASED DEFINITE INTEGRATION PROBLEMS**

21. \( \int_{0}^{\pi/2} \frac{\sin x}{\sin x + \cos x} \, dx \)
22. \( \int_{0}^{\pi/2} \frac{\cos x}{\cos x + \sin x} \, dx \)
23. \( \int_{0}^{\pi/2} \frac{\sqrt{\sin x}}{\sin x + \sqrt{\cos x}} \, dx \)
24. \( \int_{0}^{\pi/2} \frac{\sqrt{\cos x}}{\cos x + \sqrt{\sin x}} \, dx \)
25. \( \int_{0}^{\pi/2} \frac{\cos^2 x}{\cos^2 x + \sin^2 x} \, dx \)
26. \( \int_{0}^{\pi/2} \frac{\cos^n x}{\cos^n x + \sin^n x} \, dx \)
27. \( \int_{0}^{\pi/2} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} \, dx \)
28. \( \int_{0}^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} \, dx \)
29. \( \int_{0}^{\pi/2} \frac{\cos x}{\tan x + \sqrt{\cot x}} \, dx \)
30. \( \int_{0}^{\pi/2} \frac{\sqrt{\cot x}}{\cot x + \sqrt{\tan x}} \, dx \)
31. \( \int_{0}^{\pi/2} \frac{1}{1 + \tan x} \, dx \)
32. \( \int_{0}^{\pi/2} \frac{1}{1 + \cot x} \, dx \)
33. \( \int_{0}^{\pi} \log \tan x \, dx \)
34. \( \int_{0}^{\pi} \log \cot x \, dx \)
35. \( \int_{0}^{\pi/4} \log(1 + \tan \theta) \, d\theta = \frac{\pi}{8} \log(2) \)
36. \( \int_{0}^{\pi} \frac{\sin x + b \cos x}{\sin x + \cos x} \, dx \)
37. \( \int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} \, dx \)
38. \( \int_{0}^{\pi} \frac{x \sin x}{1 + \sin x} \, dx \)
39. \( \int_{0}^{\pi/4} \log(1 + \tan \theta) \, d\theta \)
40. \( \int_{0}^{\pi} 2 \sin 2x \log \tan x \, dx \)
41. \( \int_{0}^{\pi} \log \left| \frac{2 + 3 \sin x}{2 + 3 \cos x} \right| \, dx \)
2. DEFINITE INTEGRALS

QUADRATURE (AREAS)

42. Find the area of the region bounded by the curve \(2y = x^2\), the x-axis and the ordinates \(x=1\) and \(x=3\).

43. Find the area bounded by the curve \(y = \cos x\), x-axis between the limits \(x=0\) to \(x=2\pi\).

44. Find the area of the region bounded by the curve \(y = x^2\), the x-axis and the line \(x=4\).

45. Find the area included between the parabola \(y^2 = 4ax\) and its latus rectum.

46. Find the area bounded by the parabola \(y^2 = 4ax\), the x-axis and the lines \(x=1\) and \(x=4\) in the first quadrant.

47. Find the area bounded by the curve \(y^2 = 8x\), the y-axis and the lines \(y=2\) and \(y=6\).

48. Find the area of the region bounded by the curve \(y^2 = 4x\) and \(x=3\).

49. Find the area enclosed by the parabola \(y = x^2 - 8x + 15\) and the x-axis.

50. Find the area enclosed by the circle \(x^2 + y^2 = r^2\).

51. Prove that the area common to the parabolas \(y^2 = 4ax\) and \(x^2 = 4ay\) is \(\frac{16a^2}{3}\).

52. Find the area enclosed between the parabolas \(y^2 = 16x\) and \(x^2 = 16y\).

53. Find the area enclosed between the parabolas \(y^2 = 9x\) and \(x^2 = 9y\).

54. Find the area enclosed between the parabolas \(y^2 = 8x\) and \(x^2 = 8y\).

55. Find the area enclosed between the parabolas \(y^2 = x\) and \(x^2 = y\).

56. Find the area enclosed between the parabolas \(y^2 = 4x\) and \(x^2 = 4y\).

57. Find the area bounded by the curve \((x = 4 - y^2)\) and the y-axis.

58. Find the area of the region bounded by the ellipse \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\).

59. Find the area enclosed by \(9x^2 + 4y^2 = 36\).

60. Find the area enclosed between the curves \(x^2 = y\) and the straight line \(y=3x+4\).

61. Find the area of the segment cutoff from the parabola \(y^2 = 8x\) by the line \(2x-y-8=0\).

62. Find the enclosed area between the curve \(x^2 = y\) and the straight line \(y=x+2\).

63. Find the area enclosed by the curve \(y = x^2\) and the line \(2x-y+3=0\).

64. Find the area bounded by the parabola \(y^2 = x - 1\) and the line \(y=x-3\).

65. Find the area bounded by the parabola \(3x^2 = 4y\) and the line \(2y=3x+12\).

66. Find the area bounded by the parabola \(y^2 = 4x\) and the line \(x+y=3\).
2. DEFINITE INTEGRALS

VOLUMES OF SOLIDS OF REVOLUTION.

67. Find the volume of the solid generated when the area bounded by the curve \( y = x^2 - 1 \) and the x-axis.

68. Find the volume of the solid generated by rotating the portion of the parabola \( y = x^2 \) between \( x=0 \) and \( x=2 \) about x-axis.

69. Find the volume generated when the area enclosed by \( y = \sqrt{x} \), the x-axis and the ordinates \( x=1 \) and \( x=6 \) revolves about x-axis.

70. Find the volume of the solid generated when the area bounded by \( y^2 = x^3 \) and \( x=4 \) revolve about x-axis.

71. Find the volume of the parabolic generated by revolving the parabola \( y^2 = 4ax \) about x-axis from \( x=0 \) to \( x=h \).

72. Find the volume of the solid generated when the area bounded by the curve \( y = x(1-x) \) and x-axis.

73. Determine the volume of the solid generated by revolving the area bounded by the parabola \( y^2 = 8x \) cut off by its latus rectum given by \( x=2 \) about the x-axis.

74. The region of the circle \( x^2 + y^2 = 16 \) is revolved about a diameter. Find the sphere thus generated.

75. Find the volume generated by revolving about the x-axis an arch of the curve \( y = \sin x \).

76. Obtain the volume of the sphere of radius “r” using integration.

77. Find the volume of the solid formed by revolving the area enclosed by the curve \( y = x^3 \), the y-axis and the line \( y=0 \) and \( y=8 \) about y-axis.

78. Find the volume of the solid generated by revolving the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) about x-axis.

79. Find the volume of the solid generated by revolving the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) about y-axis.

80. Find the volume generated by revolution of the ellipse \( 9x^2 + 25y^2 = 225 \). About its x-axis.

81. Find the volume of the solid generated by revolving the area bounded by the ellipse \( 25x^2 + 16y^2 = 400 \) about its major axis.

82. Find the volume of the solid generated by revolving the area bounded by the ellipse \( 4x^2 + 9y^2 = 36 \) about its major axis.

83. Find the volume of the solid generated by revolving the area bounded by the ellipse \( 25x^2 + 16y^2 = 400 \) about its minor axis.

84. Find the volume of the solid generated by revolving the area bounded by the curve \( \sqrt{x} + \sqrt{y} = \sqrt{a} \), \( x=0 \) and \( y=0 \) about the x-axis.

85. Find the volume of the solid generated by rotating the area bounded by the curve \( \sqrt{x} + \sqrt{y} = 1 \) and the first quadrant about the y-axis.

86. Find the volume of the solid generated by revolving the area bounded by \( y = x^2 + 3 \) about x-axis from \( x=1 \) to \( x=3 \).
87. Find the mean value of $x^2 - 5x + 6$ between the values of ‘x’ where the expression vanishes.

88. Find the mean value of $x = y^2 - 6y + 5$ between the values of ‘x’ where the expression vanishes.

89. Find the mean value of $x^2 - 4x + 3$ between the values of ‘x’ where the expression vanishes.

90. If the mean value of $k \sin 2x + \sin x$ is ‘0’ in the interval $\left[0, \frac{\pi}{2}\right]$, then find “k”.

91. Find the mean value of $x^2 e^x$ from $x=1$ to $x=3$.

92. Find the mean value of $x^2 e^x$ from $x=1$ to $x=2$.

93. Find the mean value of $f(x) = \log x$ on $[1, e]$.

94. Find the mean value of $\frac{1}{1+x^2}$ in the interval $(0,1)$.

95. Find the mean value of $y^2 = 8x$ from $x=0$ to $x=3$.

96. Find RMS value of $xe^x$ as $x$ varies from $x=1$ to $x=3$.

97. Find the RMS value of $xe^{2x}$ over the interval $0 \leq x \leq 1$.

98. Find the RMS value of $\sqrt{\log x}$ over the range $x=1$ to $x=e$.

99. Find the RMS value of $\log x$ over the range $x=1$ to $x=e$.

100. Find the RMS value of $\sqrt{27 - 4x^2}$ from $x=0$ to $x=3$.

101. Find the RMS value of $y = \sqrt{8 - 4x^2}$ from $x=0$ to $x=2$.

102. Find the RMS value of $y = \sqrt{x}$ from $x=0$ to $x=1$.

103. Find the RMS value of $\sqrt{\log x}$ between the limits $x=e$ and $x=e^2$.

104. Determine the root mean square value of the function $y=xe^x$ in the range between $x=0$ and $x=1$.

105. Find the RMS value of the current $I=a \sin x$ over the half wave.

106. Find the RMS value of the current $I=\sin x$ over the full wave.